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I affirm that this report is my own work, and does not include any unacknowledged material taken from another source.

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Abstract

This study replicated some aspects of the syllogistic inference study by Johnson-Laird and Bara (1984), to test their 'three models' theory of solving syllogisms. The study used undergraduate participants ($n \approx 120$), in a repeated measures design, each attempting to solve syllogisms of all three types ($n = 12$: 4 of each type).

The results supported Johnson-Laird and Bara's findings that performance in solving one-model syllogisms was greater than for two-model syllogisms ($p < .01$), and that performance in solving two-model syllogisms was greater than for three model syllogisms ($p < .01$). The study did not, however, address the figural effect noted by Johnson-Laird and Bara.

Further study using this method might help to better quantify the influence of the figural effect by changing the order of presentation of syllogistic premises for the same set of problems.

A syllogism can be defined as ‘a formal deductive argument made up of a major premise, a minor premise, and a conclusion’. The word derives from the Latin *‘sullogismos’*: to infer*;* ultimately derived from the Greek root *‘logos’*: reason (Encarta, 2003).

A syllogism takes the form of:

All big, juicy steaks are meat	(premise)
No meat is eaten by vegetarians	*(*premise)
No big, juicy steaks are eaten by vegetarians	(conclusion)

Syllogistic logic was first explored by Aristotle as a means of structuring arguments to reach a logically valid conclusion. This system of predicate logic (so called because it separates the predicate of the sentence from the subject) has developed since the days of Aristotle, through various incarnations. The one thing all these systems have in common is the structure of the premises and conclusions. The four structures, or ‘moods’ have descriptive mnemonics:

All X are Y:	‘A’: universal affirmative
Some X are Y:	‘I’: particular affirmative
No X are Y:	‘E’: universal negative
Some X are not Y:	‘O’: particular negative

(Johnson-Laird & Bara, 1984)

In the traditional, probably mediaeval, system of logic the combining of premises and conclusions into ‘figures’ produced four possible structures. In this system, the major premise is the one that contains the subject (S) of the conclusion; and the minor premise is the one that contains the predicate (P) of the conclusion. The other object is the middle term (M) (Garnham & Oakhill, 1994).

M – P	P – M	M – P	P – M
S – M	M – S	M – S	M – S
S – P	S – P	S – P	S – P

(Garnham & Oakhill, 1994)

This system of analysis allows for 256 forms of syllogism: four moods for each premise; four possible combinations of premises: $4^4 = 256$. Of those 256 forms of syllogism, it was reckoned that twenty-four produced valid conclusions (Garnham & Oakhill, 1994).

Some syllogisms prove to be easier to solve than others. There have been many theories advanced about different mental mechanisms involved in solving them, which have demonstrated differing degrees of validity and reliability.

There have been three main interference-type effects described, which may affect the difficulty of solving syllogisms in the traditional manner:

1. The atmosphere hypothesis, described by Woodworth and Sells (1935), argues that reasoners will arrive at a conclusion that agrees with the ‘atmosphere’ of the premises.

   Woodworth and Sells argued that a negative premise (‘E’ or ‘O’) would create a negative atmosphere and steer the reasoner towards an erroneously negative conclusion, and that a particular premise (‘I’ or ‘O’) would create a particular atmosphere and steer the reasoner towards an erroneously particular conclusion. This hypothesis should be sustained when only one of the premises is either negative or particular.

2. The conversion effect occurs when reasoners reverse the order of subject and predicate. When solving: ‘all B are A: all B are C =>’ the reasoner may try to convert it to a more linear form: ‘all A are B: all B are C =>’.

   The solution to the second problem is more obvious: ‘all A are C’, but it is an invalid conclusion, because ‘all B are A’ is not the same as ‘all A are B’.

   Chapman and Chapman (1959) noted that conversion was valid for ‘I’ and ‘E’ type premises, which may have reinforced its apparent usefulness, but that it changed the semantic meaning of ‘A’ and ‘O’ type premises.

3. The figural effect relates to the order of presentation of the premises, the structure of the figures as discussed above, and the subsequent order of the subject and predicate in the conclusion.

A syllogism presented as:
A – B: B – C
will probably elicit a conclusion:
A – C

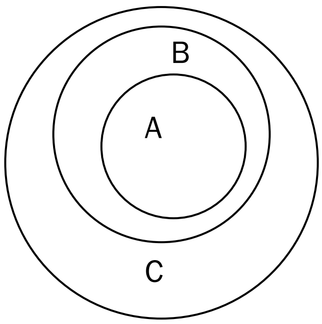
A syllogism presented as:
B – A: C – B
will probably elicit a conclusion:
C – A

(Johnson-Laird & Bara, 1984)

This may affect the ability to solve syllogisms that do not have a conclusion that is valid in both directions: A – C; and C – A.

There have been other reasons given for the varying difficulty levels of syllogisms, and explanations of how we mentally construct the problems in order to solve them.

Euler circles are a form of representing mathematical sets. Figure 1 shows a Euler circle diagram for a simple syllogism like ' all A are B: all B are C':

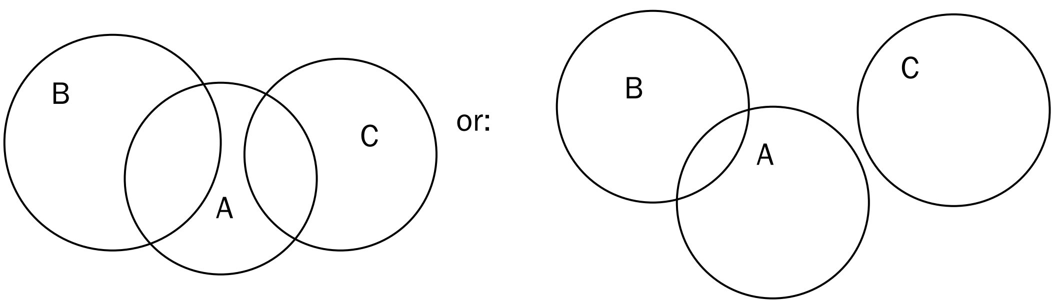


For a very simple syllogism like this, a Euler circle diagram is quite sufficient, but for more complicated problems, it does not illustrate all the possibilities.

Figure 1: a simple Euler Circle diagram

For a syllogism like 'some A are B: no C are B', there is more than one way to represent this. Figure 2 shows two examples:

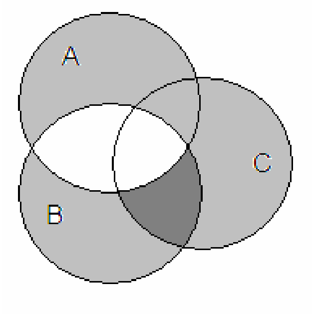
Figure 2: two Euler Circle diagrams of the same syllogism



Both these solutions are possible, but since the premises seem to define more than one conclusion, it is difficult to establish how this syllogism can have a valid conclusion (which it does: some C are not A).

Syllogisms can also be analysed using Venn diagrams, which are a slightly more useful analytical tool than Euler circles.

Figure 3: a Venn diagram



From figure 3 you can see, from the location of the dark shading, and the lack of shading, that if 'no A are B, and all C are B', then no A are C. In a similar way to Euler circles, however, there are ambiguities to this system of solving syllogisms.

Venn diagrams represent universal premises accurately and easily, but once you have particular premises, where an area of the Venn diagram may or may not contain something, they become almost as confusing as trying to solve the problem mentally. For example, 'some A are B, but no B are C' produces the diagram in figure 4: (the '+' signs indicate a particular partition).

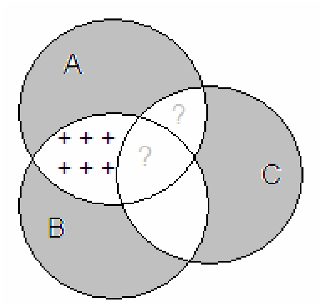


Figure 4: a more ambiguous Venn diagram

Johnson-Laird and Bara (1984) dispute the universality of the whole traditional system. They note that the traditional system only recognises conclusions that follow the 'major premise: minor premise => subject/predicate conclusion' pattern, that is:

A –B: B – C => A – C and not: A – B: B – C => C – A

Johnson-Laird and Bara approached the subject from a different angle. They rejected the model that produced 256 syllogistic forms and twenty-four valid conclusions, and started again from first principles.

They accepted the four 'mood' structures, and the four possible combinations of premises, but interpreted the possibilities in a different manner: The interaction between the three variables: A; B; C; over four possible premise structures makes sixty-four basic syllogisms: $4^3 = 64$ (Johnson-Laird & Bara, 1984).

Johnson-Laird and Bara note that 'an inference is valid if its conclusion is true in every possible interpretation of its premises'. Following this principle, twenty-seven of these syllogisms will produce a valid conclusion, because they also allow conclusions that take the form C – A, not just A – C (*ibid,* p. 4-5). They also introduced a different set of criteria for judging the difficulty of syllogisms.

Some figures of premises, like the example given above for Euler circles, can produce more than one possible conclusion. Johnson-Laird and Bara wrote that, for three variables (A; B; and C), there could be up to three possible ways to look at any given problem: three models for a single syllogism.

A syllogism that could be solved by using one model would be easier than one that required two models, and much easier than one which required three models. Figure 5 shows a one-model syllogism: 'all A are B: all B are C'.

Figure 5: a one-model syllogism

A	=	B	=	C
		(B)		(C)
				(C)

Letters in brackets are existences that are inferred (because 'all A are B' is not necessarily the same as 'all B are A'). From this notation it is easy to see that 'all A are C', or that 'some C are A'.

A two-model syllogism would require the comparison of two models of the same premises to see if there is a logically valid similarity between them. Figure 6 shows a two-model syllogism: 'all A are B: some B are not C'.

Figure 6: a two-model syllogism

Model 1				Model 2			
A	=	B		A	=	B	(C)
(A)		B	(C)	A	=	B	(C)
		(B)				(B)	
- - - - - - - -	- - - - - - - -	- - - - - - - -	- - - - - - - -	- - - - - - - -	- - - - - - - -	- - - - - - - -	- - - - - - - -
			C				C

Model 2 suggests that 'all A are C', but model 1 refutes this (dotted lines are a particular partition – some X are / are not). The only valid logical conclusion that can be drawn from this is that 'some C are not A'.

A three model syllogism requires three models of the problem to be simultaneously examined for a valid conclusion. Figure 7 shows a three-model syllogism: 'no A are B: some B are C'

Figure 7: a three-model syllogism

Model 1				Model 2				Model 3			
C	=	B		C	=	B		C	=	B	
(C)		(B)				(B)				(B)	
________	________	________	________	- - - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -
			A	(C)			A	(C)			A
			A				A	(C)			A

There is only one valid conclusion that can be drawn from these three models: 'some C are not A'.

If all syllogisms can be reduced to this three-model analysis, then we should expect to see a difference between performance levels for the three different types of problem. Johnson-Laird and Bara report as much, but they also report evidence of a figural effect, as discussed earlier.

This study will test whether performance in solving three-model syllogisms is lower than in solving two-model syllogisms, and whether performance in solving two-model syllogisms is lower than in solving one-model syllogisms.

Method

Design

This experiment uses a repeated measures design, with all of the participants performing all types of problem. The independent variable is the type of syllogism: one-model; two-model; or three-model, and the dependent variable is the score for correctly solved problems. The data collected is discrete and on a ratio scale.

Participants

Approximately 120 psychology undergraduates at the University of Strathclyde took part in the experiment as a requirement of their course. The participants are predominantly under 25 years of age, and there are more female than male students.

It is not known whether any of the participants had any training in logic, but as they are all undergraduates on a course which does not provide such training, we shall assume that the majority of participants had no prior training in logic.

A random sample of 24 sets of results was selected for statistical analysis.

Materials

The syllogisms were presented on paper (see appendix A).

Procedure

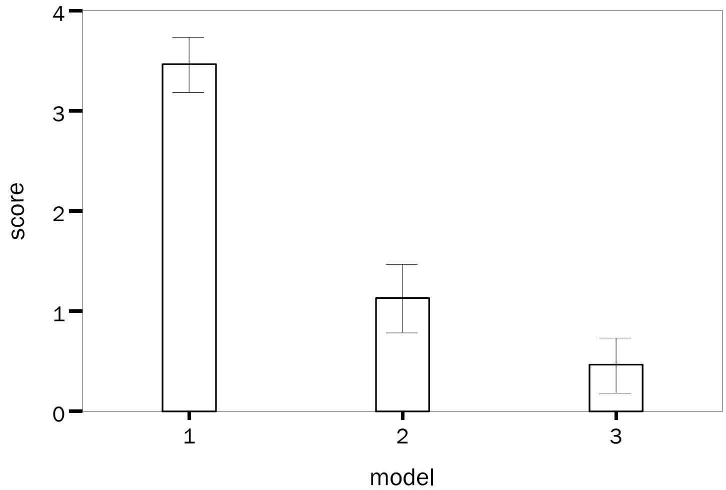
The experiment took place in a large, well-lit lecture room, with participants sitting at desks. The papers were distributed, and everyone started at the same time, working silently and independently. There was no time limit to complete the paper, but the vast majority of participants completed it within thirty minutes.

Once all the participants had finished, they swapped papers with their neighbours and the correct answers were given out. The papers were then collected and the results were collated.

Results

The mean score for one-model syllogisms was 3.46, standard deviation 0.66. The mean score for two-model syllogisms was lower, at 1.13, standard deviation 0.80. The mean score for three model syllogisms was lower again at 0.46, standard deviation 0.66. These results are shown in figure 8.

Figure 1: Mean Scores for Syllogisms



A repeated-measures, one-way ANOVA was carried out on these means, which returned a value of 140.01 with 2 degrees of freedom, which is significant at the 1% level [$F(2, 46) = 140.01$, $p < .001$] (see appendix B). Summary data for the ANOVA is shown in table 1 below.

Table 1: Summary data for one-way repeated measures ANOVA

	df	SS	MS	F	p
Model	2	119.11	59.56	140.09	<.005
Error	46	19.56	0.43		

A *post-hoc* Tukey HSD test was carried out (see appendix C), which showed that the mean score for one-model syllogisms was significantly higher than the score for two-model syllogisms ($p < .01$); the mean score for two-model syllogisms was significantly higher than the mean score for three-model syllogisms ($p < .01$); and the mean score for one-model syllogisms was significantly higher than the mean score for three-model syllogisms ($p < .01$).

Discussion

The results of this study support the findings of Johnson-Laird and Bara (1984). Performance in one-model syllogisms was significantly better than in two-model syllogisms, and performance in two-model syllogisms was significantly better than in three-model syllogisms.

This has implications for theories relating to working memory and short-term memory systems: it is obviously more difficult to hold and compare three mental models of a problem than two models or one model. It may also have implications for the way we deal with other problem-solving tasks: syllogisms are not the only problems that spring from a set of premises.

Although Johnson-Laird and Bara did report a difference caused by the figural effect, even without this factor being added in to this study, the results remain significant.

There is perhaps scope for further research into exactly how significant the figural effect is. The test could be re-administered with half of the participants having the order of presentation of the premises reversed, in a similar way to the way you would counter-balance a questionnaire. This might show the extent of the figural effect, since half of the problems presented would conform to the classical model:

A – B: B – C => A – C

while the other half would conform to the 'counter-intuitive' model:

A – B: B – C => C – A

This study has replicated the most significant findings of Johnson-Laird and Bara (1984), and supports their findings that people undertake syllogistic reasoning by building between one and three mental models of the problem in order to assess the validity or otherwise of their conclusions.

Performance in solving one-model syllogisms is greater than in solving two-model syllogisms, and performance in solving two-model syllogisms is greater than in solving three-model syllogisms.

Further study might indicate the extent to which the figural effect plays a part in the performance differences when solving syllogisms.

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Appendix A: Syllogisms

1. all A are B
all B are C
Conclusion: all A are C
or: Some C are A

Type: one-model

2. some A are B
some B are C
Conclusion: no valid conclusion

Type: two-model

3. no A are B
all B are C
Conclusion: some C are not A

Type: three-model

4. some A are not B
all B are C
Conclusion: no valid conclusion

Type: two-model

5. all B are A
all C are B
Conclusion: all C are A
or: some A are C

Type: one-model

6. no B are A
some C are B
Conclusion: some C are not A

Type: three-model

7. all A are B
no C are B
Conclusion: no A are C
or: no C are A

Type: one-model

8. some A are not B
all C are B
Conclusion: some A are not C

Type: two-model

9. some A are B
no C are B
Conclusion: some A are not C

Type: three-model

10. some B are A
all B are C
Conclusion: some A are C
or: some C are A

Type: one-model

11. no B are A
all B are C
Conclusion: some C are not A

Type: three-model

12. some B are not A
no B are C
Conclusion: no valid conclusion

Type: two-model

Appendix B: ANOVA

Appendix C: Tukey HSD

$$T = q_k \sqrt{\frac{MSError}{n}}$$

q_k for 3 means, 46 df (Error), (p = .05) = 3.44

q_k for 3 means, 46 df (Error), (p = .01) = 4.37

p = .05

$$T = 3.44\sqrt{\frac{0.425}{24}}$$

T = 3.44 x 0.133

T = 0.458, p = 0.05

p = .01

$$T = 4.37\sqrt{\frac{0.425}{24}}$$

T = 4.37 x 0.133

T = 0.581, p = 0.01

Differences between mean scores

Mean score for one-model syllogisms: 3.46

Mean score for two-model syllogisms: 1.13

Mean score for three-model syllogisms: 0.46

	one-model	two-model	three-model
one-model	---	2.33 (p<.01)	3.00 (p<.01)
two-model		---	0.87 (p<.01)
three-model			---